Section 1.6

• Absolute Value Equations

$|-2|$ ← What does that mean?

↳ 2 → But why?

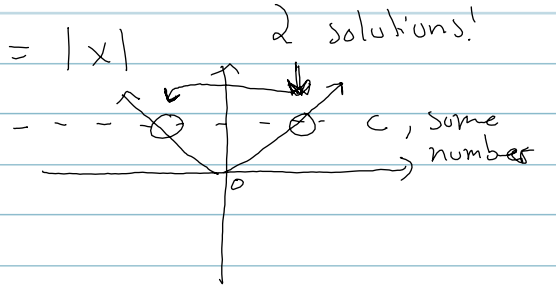
↳ "Absolute Value" is interested only in the distance from a number and 0. (-2) is 2 units away from 0 on a number line; thus, its absolute value is 2

- $|x| = 5$

5 and −5

Both are good solutions. ⇒ $x = 5$ or $x = -5$

$y = |x|$



- $|x+3| = 4$

$x+3 = 4$ and $x+3 = -4$

$x = 1$ $x = -7$

- $2|x-7|+3 = 7$

$-3 \quad -3$

$2|x-7| = 4$

$|x-7| = 2$

NO

distribution, absolute values act as walls. So, get rid of everything around them!

$x-7 = 2$ and $x-7 = -2$

$x = 9$ $x = 5$

★ $|x+2| = -3$ ← Look at this! ★

$|$ hmm...

$x+2 = -3$ → $x = -5$ — $x+2 = 3$ → $x = 1$ — WRONG!

★ Can an absolute value ever yield a negative number? Confused? Just check!

$|-5+2| = -3$

$|-3| = -3$

$3 \neq -3$ (X)

$|1+2| = -3$

$|3| = -3$

$3 \neq -3$ (X)

★ Just watch out!

○ Intersection ⇒ ∩ ⇒ AND ⇒ "What is in common?"

○ Union ⇒ ∪ ⇒ OR ⇒ Everything!

★ $A = \{1, 2, 3, 4, 5\}$

$B = \{2, 4, 6, 8\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$

$A \cap B = \{2, 4\}$

Absolute Values - Less than - ANDS / Intersections / ∩

★ $|x-3| < 2$ ← watch out; you are still moving the sign over!

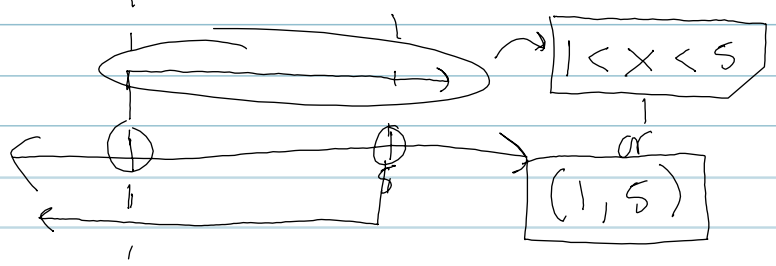
$x-3 \leq 2$ $\quad$ $x-3 \not\leq -2$

This is the solution!

$x-3 < 2$

$x < 5$ ∩ $x-3 > -2$

$x > 1$ ⇒



• $3|x-1|+4 \leq 7$

$3|x-1| \leq 3$

$|x-1| \leq 1$

$x-1 \leq 1$ (n) $x-1 \geq -1$

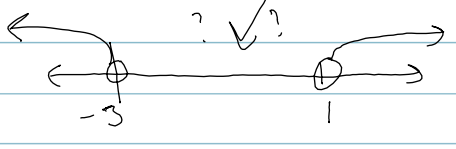
$x \leq 2 \longrightarrow x \geq 0 \longrightarrow 0 \leq x \leq 2$

↓ or → $[0, 2]$

☆ $|x+1| < -2$

$x+1 < -2$ (n) $x+1 > 2$

$x < -3$ $x > 1$



No solution. But why?

Well...

$|x+1| < -2$

Can an absolute value ever be less than -2? or even 0? Nope! So...

∅ No Solution!

Absolute Value - Greater Than

└ Union → ∪ → Everything!

☆ $|x-3| > 5$

$x-3 > 5$ (∪) $x-3 < -5$

$x > 8$ (∪) $x < -2$ ⟹ $x > 8$ or $x < -2$

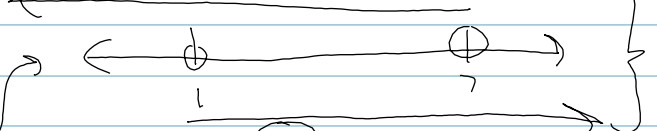
$(-\infty, -2) \cup (8, \infty)$

☆☆ $|x-4| > -3$

$x-4 > -3$ (∪) $x-4 < 3$

$x > 1$ (∪) $x < 7$

hmm...



$(-\infty, \infty)$!!

Remember, (∪) means OR!

As long as one restriction is satisfied, it's true!

$|x-4| > -3$ $\Rightarrow$ ALL positive numbers are greater than -3, or even 0! So, the solution is All Real numbers (or $x \in \mathbb{R}$, to be fancy!)